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Radiative transfer in a coating on a rectangular corner: allowance for shadowing

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Abstract—Exact integral equations for the radiative transfer in a coating on the exterior of a rectangular corner are derived. The model includes shadowing effects of the substrate and diffuse reflections at the substrate surfaces. The integral equations for the source function and the interface intensity are solved by a Gaussian quadrature integration method. The directional emittances of the coating at various locations are evaluated. Influence of substrate properties, scattering albedos and optical thicknesses on the emittance is examined.

INTRODUCTION

THE PROBLEM of radiative heat transfer in a participating medium has become increasingly important in numerous situations. They include, among others, coatings for temperature control of surfaces, thermal protection, energy conversion [1] and cryogenics [2]. A coating is usually considered as a semi-transparent medium with a substrate. Since the apparent properties of coatings are of practical importance, numerous approximate and exact methods have been applied to study the properties. These methods include the $P-1$ approximation [3], the direct integration of Beer's law [4], the Monte Carlo method [5], the application of Chandrasekhar's X- and Y-functions [6], the normal-mode expansion [7], the exponential kernel approximation [8], the discrete-ordinate method [9], and the Gaussian quadrature integration method of integral formulation [10]. Although the effects of two-dimensional geometries and various boundaries have been examined, most of the reported analysis is limited to very simple geometries. In a large number of technological systems, coatings are applied to non-planar substrates. A convex substrate may cut off the path of radiative transfer in a coating, while radiation leaving a coating on a concave substrate may re-enter the coating. Such geometries result in the complexity of analysis.

In this work, radiative transfer in a coating on the exterior of a rectangular corner is considered. The coating emits, absorbs and isotropically scatters thermal radiation; the interface between the coating and the substrate is opaque and diffuse. The difficulty of analysis resulting from shadowing effects of the non-planar boundaries can be overcome by the adaptation of the integral formulation which has been applied to simple geometries [11]. Coupled integral equations for the source function in the medium and the unknown intensity at the diffusely reflecting interface are developed. The advantage in employing the integral formulation is the reduction of independent variables [12]. Similar integral formulations can be developed for radiative transfer in coatings on the other non-planar surfaces, such as obtuse-angle corners and streamline surfaces. The physical situations might simulate radiative transfer in coatings and tiles on a spacecraft. Then, the Gaussian quadrature integration method is used to solve the coupled integral equations. To illustrate the application of the present procedure, we apply the present formulation to find the emittance of the coating. The effects of substrates, scattering albedos and optical thicknesses on the emittance are discussed.

ANALYSIS

Two-dimensional radiative transfer in a coating on the exterior of a rectangular corner is considered. The coating is assumed to be an absorbing, emitting and isotropically scattering medium which is homogeneous and has a refractive index of unity. Figure 1 presents the geometry and the coordinate system. The thickness of the coating is assumed to be constant, that is, $b = c$. This results in the symmetry of the radiation field with respect to the surface $y = z$. The interface between the coating and the substrate is assumed to be opaque and diffuse and to have constant emittance ε_i and reflectance ρ_i, while the other two interfaces are assumed to be free boundaries with zero incident radiation. Figure 1 also shows that an observer at point P can not see the medium above the surface AB. This is due to the shadowing effects of the substrate. The problem of interest here is to examine the shadowing effects and to determine the distribution of radiant energy emerging from the coating at free boundaries.

The equation of radiative transfer can be expressed as

$$\frac{\mathrm{d}I(s,\mathbf{\Omega})}{\mathrm{d}s} + I(s,\mathbf{\Omega}) = S(s) \tag{1}$$

NOMENCLATURE

- b optical thickness of the coating on $y = b$
- c optical thickness of the coating on $z = c$
- d_1, d_2, d_3 optical distance defined in equations (11)–(13), respectively
- d_4 optical distance defined in equation (15)
- I dimensionless radiation intensity
- I_i inward dimensionless radiation intensity at the interfaces
- I_{bi} dimensionless Planck's black body radiation intensity at the substrate surface
- $\mathbf{r}$ optical position vector
- $\mathbf{r}_i$ optical position vector at interfaces
- s optical distance along a line of sight
- S dimensionless source function
- S_n generalized exponential integral function defined in equation (10)
- u upper bound of integration defined in equation (14)
- x, y, z optical variables
- y_2 optical width of a two-dimensional region.

Greek symbols

- ε directional emittance of the coating
- ε_i substrate emittance
- ε_n normal emittance of the coating
- θ polar angle
- θ_e $\pi-\theta$
- μ_e $\cos\theta_e$
- ρ_i substrate reflectance
- ϕ azimuthal angle
- ω scattering albedo
- Ω solid angle
- $\mathbf{\Omega}$ unit directional vector.

where I is the dimensionless radiation intensity defined as the radiation intensity divided by Planck's black body radiation intensity of the medium, s the optical distance defined as the production of the extinction coefficient and the geometrical distance along a 'line of sight', $\mathbf{\Omega}$ the direction unit vector determined by the polar angle θ and azimuthal angle ϕ, and S the dimensionless source function defined as the source function divided by Planck's black body radiation intensity of the medium. Here, S can be expressed as

$$S(s) = (1-\omega)+\frac{\omega}{4\pi}\int_{4\pi} I(s,\mathbf{\Omega}')\,d\Omega' \quad (2)$$

where ω is the albedo defined as the scattering coefficient divided by the extinction coefficient, $d\Omega'$ is an element of solid angle. Besides, the subscript v which denotes the spectrally dependent properties of

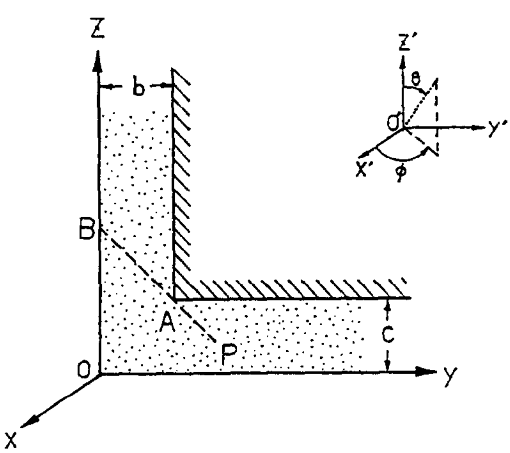

FIG. 1. Geometry and coordinates.

the medium and the boundaries is omitted to simplify the mathematical expressions. The boundary conditions for the present problem are

$$I_i(\mathbf{r}_i) = 0 \text{ for all inward directions on the surfaces } y = 0 \text{ and } z = 0 \quad (3)$$

$$I_i(\mathbf{r}_i) = \varepsilon_i I_{bi}(\mathbf{r}_i) + (\rho_i/\pi)\int_{\phi'=0}^{2\pi}\int_{\theta'=0}^{\pi/2} I(\mathbf{r}_i,\theta',\phi') \times \cos\theta' \sin\theta'\,d\theta'\,d\phi' \text{ for all inward directions on the surface } z = c \text{ with } y > b \quad (4)$$

where I_i denotes the dimensionless radiation intensity defined as the interface radiative intensity in the inward direction divided by Planck's black body radiation intensity of the medium, $\mathbf{r}_i$ the optical position vector defined as the product of the extinction coefficient and the geometric position vector on the substrate surface or the free boundary, and I_{bi} the dimensionless radiation intensity defined as Planck's black body radiation intensity of the substrate divided by that of the medium.

To obtain the integral formulation of the present problem, one can follow the procedure similar to that of refs. [10–13]. First, the dimensionless formal solution of equation (1) may be expressed as

$$I(\mathbf{r},\mathbf{\Omega}) = I_i(\mathbf{r}_i)\,e^{-|\mathbf{r}-\mathbf{r}_i|}+\int_{\mathbf{r}_i}^{\mathbf{r}} S(\mathbf{r}')\,e^{-|\mathbf{r}-\mathbf{r}'|}\,ds' \quad (5)$$

where $\mathbf{r}$ is the optical position vector defined as the product of the extinction coefficient and the geometric position vector and ds' is an infinitesimal optical distance in the direction $\mathbf{\Omega}$. Here

$$\mathbf{\Omega} = (\mathbf{r}-\mathbf{r}_i)/|\mathbf{r}-\mathbf{r}_i| = (\mathbf{r}-\mathbf{r}')/|\mathbf{r}-\mathbf{r}'|. \quad (6)$$

Next, substituting equation (5) into equations (2) and (4) and transforming the integrals over solid angle in the resulting equations into surface integrals and volume integrals produces the integral equations for S and I_i. Finally, since the medium in the $\pm x$-directions is unbounded, the integral equations for S and I_i can be reduced to two-dimensional forms:

$$\begin{aligned} S(y,z) &= (1-\omega) \\ &+\frac{\omega}{4}\int_{z'=0}^{c}\int_{y'=z'}^{\infty} S(y',z')\frac{S_1(d_1)}{d_1}\,dy'\,dz' \\ &+\frac{\omega}{4}\int_{y'=0}^{b}\int_{z'=y'}^{\infty} S(y',z')\frac{S_1(d_1)}{d_1}\,dz'\,dy' \\ &+\frac{\omega}{4}\int_{b}^{\infty} I_i(y',c)\frac{S_2(d_2)}{d_2^2}(c-z)\,dy' \\ &+\frac{\omega}{4}\int_{c}^{\infty} I_i(b,z')\frac{S_2(d_3)}{d_3^2}(b-y)\,dz' \end{aligned}$$

$$\text{in } z \leqslant y < b \quad (7)$$

$$\begin{aligned} S(y,z) &= (1-\omega) \\ &+\frac{\omega}{4}\int_{z'=0}^{c}\int_{y'=z'}^{\infty} S(y',z')\frac{S_1(d_1)}{d_1}\,dy'\,dz' \\ &+\frac{\omega}{4}\int_{y'=0}^{b}\int_{z'=y'}^{u} S(y',z')\frac{S_1(d_1)}{d_1}\,dz'\,dy' \\ &+\frac{\omega}{4}\int_{b}^{\infty} I_i(y',c)\frac{S_2(d_2)}{d_2^2}(c-z)\,dy' \end{aligned}$$

$$\text{in } y \geqslant b \quad (8)$$

$$\begin{aligned} I_i(y,c) &= \varepsilon_i I_{bi} \\ &+\rho_i\int_{z'=0}^{c}\int_{y'=z'}^{\infty} S(y',z')\frac{S_2(d_4)}{d_4^2}(c-z')\,dy'\,dz' \\ &+\rho_i\int_{y'=0}^{b}\int_{z'=y'}^{c} S(y',z')\frac{S_2(d_4)}{d_4^2}(c-z')\,dz'\,dy' \end{aligned} \quad (9)$$

where

$$S_n(r) = \frac{2}{\pi}\int_1^{\infty}\frac{e^{-rt}}{t^n(t^2-1)^{1/2}}\,dt \quad \text{with } n = 1,2,\ldots \quad (10)$$

$$d_1 = ((y-y')^2+(z-z')^2)^{1/2} \quad (11)$$

$$d_2 = ((y-y')^2+(z-c)^2)^{1/2} \quad (12)$$

$$d_3 = ((y-b)^2+(z-z')^2)^{1/2} \quad (13)$$

$$u = z+(y-y')(c-z)/(y-b) \quad (14)$$

$$d_4 = ((y-y')^2+(c-z')^2)^{1/2}. \quad (15)$$

The second integral in equation (8) is bounded in the z-direction and only one substrate surface has a contribution to the source function because of the shadowing effects. The symmetry of the radiation field gives the relations

$$I_i(b,z) = I_i(y,c) \quad \text{as} \quad y = z \quad (16)$$

and

$$S(y,z) = S(z,y). \quad (17)$$

Using the symmetric relations (16) and (17), we only need to solve the integral equations (7)–(9) for $y \geqslant z$. The singularity in the kernel of the integral equations can be removed by the technique employed by Crosbie and Schrenker [14]. Some integral terms in equations (7)–(9) can be evaluated analytically at the wall; effective techniques for evaluating those integral terms were also developed by Crosbie and Schrenker [14]. In practice, the radiation field becomes one-dimensional at some locations far from the corner. Thus, two-dimensional integral equations (7)–(9) are applied to a finite domain bounded by a finite distance y_2 from the corner and the radiation field for $y > y_2$ are assumed to be one-dimensional [10]. Then, the integral equations are discretized by using numerical integration formula [10, 14]. We employ the Gaussian quadrature in the present expressions. The resultant algebraic equations are solved by Gaussian elimination with pivoting.

If $\mathbf{r}$ is a location on the free boundary and $\mathbf{\Omega}$ is in an outward direction, the expression for the dimensionless radiation intensity, equation (5), is also the expression for the directional emittance from the coating. Thus after obtaining the values of $S(y,z)$ and $I_i(y,c)$, one can use equation (5) to find the directional emittance. Detailed expressions for the directional emittance on four typical view planes are given in the Appendix.

RESULTS AND DISCUSSION

To illustrate the application of the present integral formulation, the emittance of the coating is evaluated from the solutions of the integral equations. In this work we take the optical width of the two-dimensional region to be 10.0, because the total radiation contributed by the medium at a distance greater than 10.0 is less than 10^{-3} for $\omega \leqslant 0.9$ [10]. Because of symmetry, we need only examine the emittance from the surface $z = 0$. The directional emittances are evaluated for various surface locations and optical parameters.

Some comparisons are made in Figs. 2–4 to show the influence of the substrate on the emittance of a coating. Two cases have been examined: a coating with a non-emitting substrate and a coating with an emitting substrate. Figure 2 shows that, in general, the normal emittance (ε_n) increases with the substrate reflectance of a non-emitting substrate as expected. A comparison of the present results with Bobco's results

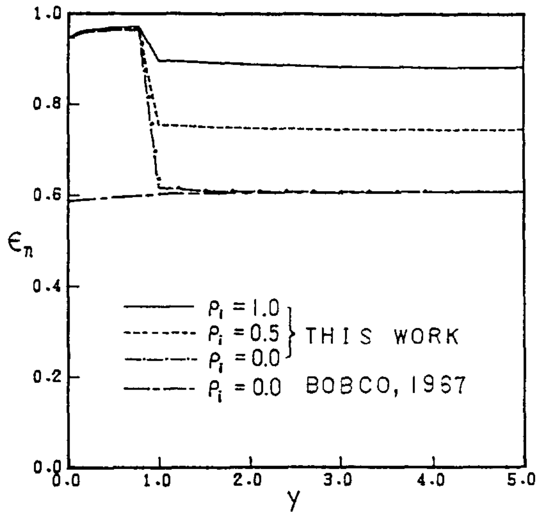

FIG. 2. Influence of the substrate on the normal emittance: $b = c = 1.0$, $\omega = 0.1$, $I_{bi} = 0.0$.

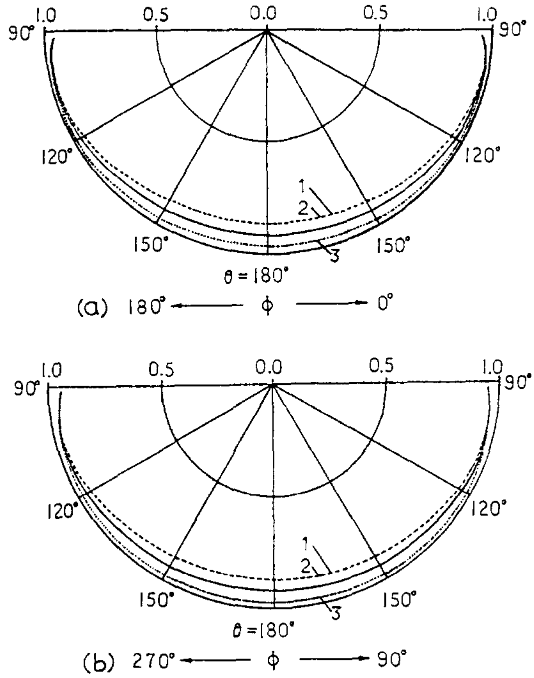

FIG. 4. Influence of the substrate on the directional emittance at $y = 4.5$: $b = c = 1.0$, $\omega = 0.1$, $I_{bi} = 1.0$ (1, $\rho_i = 1.0$, $\varepsilon_i = 0.0$; 2, $\rho_i = 0.5$, $\varepsilon_i = 0.5$; 3, $\rho_i = 0.0$, $\varepsilon_i = 1.0$).

[3] shows that the radiation at locations far from the corner approaches that in a semi-infinite slab. The agreement for limiting cases shows the validity of the present method of solution. The quick increase of the normal emittance from the surface $z = 0$ at $y = b^-$ for the low scattering case ($\omega = 0.1$) is due to the corresponding increase of the optical thickness in the z-direction at $y = b^-$. The higher values of normal emittance at $y < b$ are not sensitive to the substrate reflectance, because at $y < b$ the medium extends to infinity and no radiation from the substrate going straight to the direction normal to the surface $z = 0$. The effect of the substrate properties on the normal emittance from the surface $z = 0$ at $y < b$ is due to multiple scattering if the coating is sufficiently thick.

The directional emittance from the locations $y = 0.3$ and 4.5 of a coating with an emitting and reflecting substrate is presented in Figs. 3 and 4, respectively. In general, the intensity of radiation leaving the coating strongly depends on the direction and the location of the emerging radiation, because they determine the optical path length and the starting point of the line of sight in the coating. At a location close to the corner, say $y = 0.3$, on the view planes $\phi = 0$ and π the starting point of a line of sight is at infinity, on the view plane $\phi = \pi/2$ the starting point of a line of sight is on the free boundary, and on the view plane $\phi = 3\pi/2$ the starting point of the line of sight is on the substrate. Hence, the influence of the substrate on the directional emittance (ε) on the view planes $\phi = 0, \pi/2, \pi$ is very weak and the influence of the substrate on ε on the view plane $\phi = 3\pi/2$ is strong, as shown in Fig. 3. Besides, when θ approaches $\pi/2$ and π, because of the increase of the optical path length for the directional emittance at $y = 0.3$ on view plane $\phi = 3\pi/2$, the influence of the substrate on the emittance decreases. For the directional emittance leaving a location far from the corner, say $y = 4.5$,

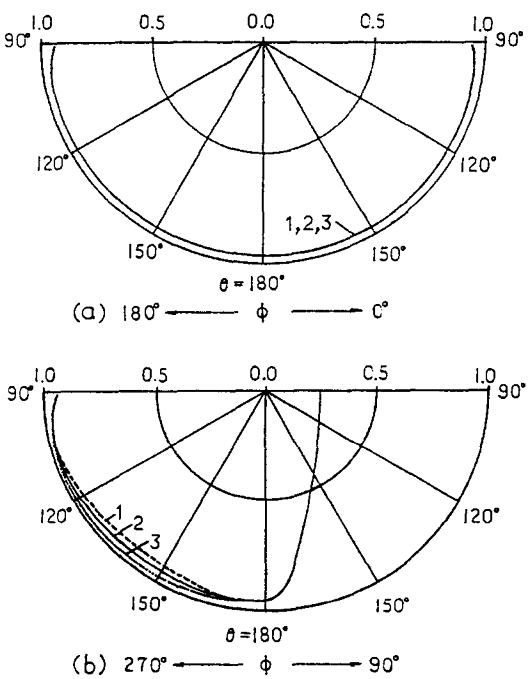

FIG. 3. Influence of the substrate on the directional emittance at $y = 0.3$: $b = c = 1.0$, $\omega = 0.1$, $I_{bi} = 1.0$ (1, $\rho_i = 1.0$, $\varepsilon_i = 0.0$; 2, $\rho_i = 0.5$, $\varepsilon_i = 0.5$; 3, $\rho_i = 0.0$, $\varepsilon_i = 1.0$).

the optical path length is almost independent of the azimuthal angle and the starting points of most lines of sight are on the substrate. Hence, the dependence of the directional emittance on the azimuthal angle is very weak and the influence of the substrate on the directional emittance is strong except when θ approaches $\pi/2$, as shown in Fig. 4. In summary, the dependence of the directional emittance on the azimuthal angle becomes larger as the point studied moves closer to the corner. This result may be called the corner effects.

The directional emittances on various view planes at $y = 0.3$, 1.0 and 4.5 are shown in Fig. 5. The magnitude of directional emittance in a fixed direction increases as the location studied moves away from the corner, as shown in Fig. 5. The effects of scattering inferred by comparing Figs. 3, 4 (where $\omega = 0.1$) and 5 (where $\omega = 0.9$) include: (1) the emittance decreases with an increase in the scattering albedo, (2) the variation of the directional emittance along the y-direction is more pronounced for a coating with a larger albedo and emitting substrates, and (3) the high scattering cases have stronger directional characters than the low scattering cases.

When the medium and the substrate are at the same temperature ($I_{bi} = 1.0$), the normal emittance increases with the increase of the optical thickness, as shown in Fig. 6. The influence of optical thicknesses is more pronounced at $y < b$, where the medium extends to infinity in the z-direction. The decrease of

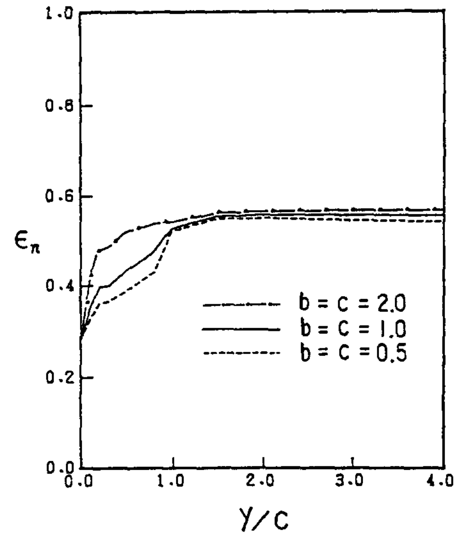

FIG. 6. Influence of optical thicknesses: $\omega = 0.9$, $\varepsilon_i = 0.5$, $\rho_i = 0.5$, $I_{bi} = 1.0$.

the normal emittance around the corner ($0 < y < b$) is due to the outward scattering through the side wall $y = 0$ for the high scattering case ($\omega = 0.9$). If the substrate temperature is far higher than the medium temperature, the effective emittance of the coating–substrate combination increases with the decrease of the optical thickness and the decrease of the scattering albedo.

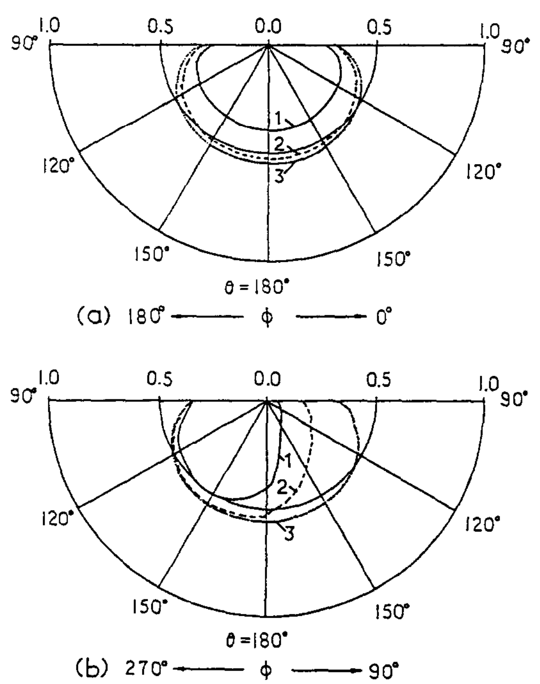

FIG. 5. Directional emittance for various locations and azimuthal angles: $b = c = 1.0$, $\omega = 0.9$, $\varepsilon_i = 0.5$, $\rho_i = 0.5$, $I_{bi} = 1.0$ (1, $y = 0.3$; 2, $y = 1.0$; 3, $y = 4.5$).

CONCLUSION

Exact integral expressions for the radiative transfer in a coating on a quarter space are derived. The model includes scattering in the medium and diffuse reflections at the substrate surfaces. The integral equations of the source function and the diffuse intensity at the substrate surface are solved by the Gaussian quadrature integration method. The influence of the coating's albedo, geometry and substrate properties on the coating's apparent emittance is demonstrated by substituting the solutions of the integral equations into the exact expressions of the directional emittance. Because of the shadowing effects, both the emittance and the influence of the physical properties on the emittance show strong two-dimensional characters and angular dependence as the location studied is close to the corner. Since the integral expression for the divergence of the radiative flux can be developed by a procedure similar to that of this work, we can apply the extension of this work to estimate the emittance from non-isothermal coatings, in which the temperature and black body emissive power distribution are determined by the principle of coupled conduction–radiation heat transfer.

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APPENDIX: EXPRESSIONS FOR DIRECTIONAL EMITTANCE

For convenience in the following expressions, we denote $\pi-\theta$ by θ_e and $\cos\theta_e$ by μ_e.

On the view plane perpendicular to both $y = 0$ and $z = 0$

$$\varepsilon(y,0,\theta,\phi) = I_i[b,(b-y)/\tan\theta_e]\exp[-(b-y)/\sin\theta_e] + \frac{1}{\mu_e}\int_0^{(b-y)\tan\theta_e} S(y+z'\tan\theta_e, z')\exp(-z'/\mu_e)\,dz',$$

$$y < b, \phi = 3\pi/2, \text{ and } 0 \leqslant \theta_e \leqslant \tan^{-1}[(b-y)/c] \quad \text{(A1)}$$

$$\varepsilon(y,0,\theta,\phi) = I_i(y+c\tan\theta_e, c)\exp(-c/\mu_e) + \frac{1}{\mu_e}\int_0^{c} S(y+z'\tan\theta_e, z')\exp(-z'/\mu_e)\,dz',$$

$$y < b, \phi = 3\pi/2, \text{ and } \tan^{-1}[(b-y)/c] < \theta_e \leqslant \pi/2 \quad \text{(A2)}$$

$$\varepsilon(y,0,\theta,\phi) = \frac{1}{\mu_e}\int_0^{y/\tan\theta_e} S(y-z'\tan\theta_e, z')\exp(-z'/\mu_e)\,dz',$$

$$y < b, \phi = \pi/2, \text{ and } 0 \leqslant \theta_e \leqslant \pi/2 \quad \text{(A3)}$$

$$\varepsilon(y,0,\theta,\phi) = I_i(y+c\tan\theta_e, c)\exp(-c/\mu_e) + \frac{1}{\mu_e}\int_0^{c} S(y+z'\tan\theta_e, z')\exp(-z'/\mu_e)\,dz',$$

$$y \geqslant b, \phi = 3\pi/2, \text{ and } 0 \leqslant \theta_e \leqslant \pi/2 \quad \text{(A4)}$$

$$\varepsilon(y,0,\theta,\phi) = I_i(y-c\tan\theta_e, c)\exp(-c/\mu_e) + \frac{1}{\mu_e}\int_0^{c} S(y-z'\tan\theta_e, z')\exp(-z'/\mu_e)\,dz',$$

$$y \geqslant b, \phi = \pi/2, \text{ and } 0 \leqslant \theta_e \leqslant \tan^{-1}[(y-b)/c] \quad \text{(A5)}$$

$$\varepsilon(y,0,\theta,\phi) = \frac{1}{\mu_e}\int_0^{y/\tan\theta_e} S(y-z'\tan\theta_e, z')\exp(-z'/\mu_e)\,dz',$$

$$y \geqslant b, \phi = \pi/2, \text{ and } \tan^{-1}[(y-b)/c] < \theta_e \leqslant \pi/2. \quad \text{(A6)}$$

On the view plane perpendicular to $z = 0$ and parallel to $y = 0$

$$\varepsilon(y,0,\theta,\phi) = \frac{1}{\mu_e}\int_0^{\infty} S(y,z')\exp(-z'/\mu_e)\,dz',$$

$$y < b, \phi = 0 \text{ or } \pi, \text{ and } 0 \leqslant \theta_e \leqslant \pi/2 \quad \text{(A7)}$$

$$\varepsilon(y,0,\theta,\phi) = I_i(y,c)\exp(-c/\mu_e) + \frac{1}{\mu_e}\int_0^{c} S(y,z')\exp(-z'/\mu_e)\,dz',$$

$$y \geqslant b, \phi = 0 \text{ or } \pi, \text{ and } 0 \leqslant \theta_e \leqslant \pi/2. \quad \text{(A8)}$$

TRANSFERT RADIATIF D'UN REVETEMENT D'UN COIN RECTANGULAIRE: PRISE EN COMPTE DE L'OMBRAGE

Résumé—On établit les équations intégrales exactes du transfert radiatif pour un revêtement à l'extérieur d'un dièdre rectangulaire. Le modèle inclut les effets d'ombrage sur le substrat. Les équations intégrales pour la fonction source et la luminance à l'interface sont résolues par une méthode gaussienne d'intégration en quadrature. Les émittances du revêtement en différents endroits sont évaluées. On examine l'influence des propriétés du substrat, des albedos et des épaisseurs optiques sur l'émittance.

WÄRMEÜBERTRAGUNG DURCH STRAHLUNG IN EINER BESCHICHTUNG IN EINER RECHTWINKLIGEN ECKE—BERÜCKSICHTIGUNG DER ABSCHATTUNG

Zusammenfassung—Es werden die exakten Integralgleichungen für die Wärmeübertragung durch Strahlung in einer Beschichtung außen an einer rechtwinkligen Ecke formuliert. Das Modell berücksichtigt Abschattungseffekte des Substrats sowie diffuse Reflexion an der Substratoberfläche. Die Integralgleichungen für die Quellfunktion und die Strahlungsstärke an der Trennfläche werden mit einer Gauß'schen quadratischen Integrationsmethode gelöst. Die gerichteten Emissionsgrade des Substrats werden an verschiedenen Stellen berechnet. Es wird der Einfluß der Stoffeigenschaften des Substrats, eines streuenden Albedos und der optischen Dicke auf den Emissionsgrad untersucht.

РАДИАЦИОННЫЙ ПЕРЕНОС В ПОКРЫТИИ НА ПРЯМОМ УГЛЕ: УЧЕТ ЗАТЕНЕНИЯ

Аннотация—Выводятся точные интегральные уравнения для радиационного переноса в покрытии на внешней поверхности прямого угла. Модель включает эффекты затенения подложки и диффузные отражения у ее поверхностей. Интегральные уравнения для функции источника и интенсивности на границе раздела решаются методом Гаусса. Оценивается направленная светимость покрытия при различных расположениях. Исследуется влияние свойств подложки, рассеивающих альбедо и оптических толщин на лучеиспускательную способность.